

Starobinsky inflation from new-minimal supergravity

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Abstract

In the new-minimal supergravity formulation we present the embedding of the $R + R^2$ Starobinsky model of inflation. Starting from the superspace action we perform the projection to component fields and identify the Starobinsky model in the bosonic sector. Since there exist no other scalar fields, this is by construction a single field model. This higher curvature supergravity also gives rise to a propagating massive vector. Finally we comment on the issues of higher order corrections and initial conditions.

1 Introduction and discussion

The Planck collaboration's constraints on inflation [1, 2] have restricted the inflationary models to those which are characterized by a plateau potential for the inflaton field. More specifically, if the perturbations during inflation [3] are originated by the same field driving inflation, these restrictions can be quantified by the following constraints on the spectral index: $n_s = 0.9655 \pm 0.0062$ and the tensor-to-scalar ratio: $r < 0.12$. A model which lies in the heart of the data is the pure gravitational Starobinsky model [4]

$$e^{-1}\mathcal{L} = \frac{M_P^2}{2}R + \frac{M_P^2}{12m^2}R^2, \quad (1)$$

which in the Einstein frame describes a real scalar field minimally coupled to gravitation, with a scalar potential given by

$$V_{R^2} = \frac{3}{4}m^2M_P^2 \left(1 - e^{-\sqrt{\frac{2}{3}}\phi/M_P}\right)^2. \quad (2)$$

This model gives $n_s - 1 \simeq -2/N$ and $r \simeq 12/N^2$. The Planck data constrain $m \simeq 1.3 \times 10^{-5}M_P$. Discussions on the generic properties of models with plateau potentials can be found in [5, 6].

Supergravity [7, 8], as the low energy limit of string theory, is essentially the appropriate framework to study high energy gravitational phenomena like inflation. The minimal 4D N=1 supergravity multiplet contains as physical fields the graviton, with 6 bosonic off-shell degrees of freedom, and the gravitino, with 12 off-shell fermionic degrees of freedom. The remaining 6 off-shell bosonic degrees of freedom are auxiliary and can be distributed as follows

- Old-minimal supergravity auxiliary fields sector [9,10]: a complex scalar M (2 DOF) and a real vector b_m (4 DOF).
- New-minimal supergravity auxiliary fields sector [11]: a gauge vector for the R-symmetry A_m (3 DOF) and a gauge two-form B_{mn} (3 DOF).

The existence of different minimal supergravities can be understood as arising from different solutions to the superspace Bianchi identities, or as different choice of appropriate Wess-Zumino gauge for the gravitational multiplet, or also as originating from the different compensating multiplets that break the underlying superconformal theory to super-Poincaré. The underlying dualities among the compensating multiplets survive the gauge fixing and lead to equivalent couplings to matter [12], but break down as soon as higher curvature terms are introduced. Here we present the embedding of the Starobinsky model of inflation in new-minimal supergravity [13]. This higher curvature supergravity is on-shell equivalent to standard supergravity coupled to a massive vector multiplet [13–16].

The Starobinsky model of inflation is nevertheless accompanied by a series of open issues. The first concerns the existence of possible higher order curvature corrections. For example the R^4 terms [13]. As we will see, these terms spoil the plateau of the scalar potential and if they are relatively large Starobinsky inflation does not take place. Therefore, one requires a hierarchy to hold during inflation

$$\frac{M_P^2}{m^2} R_{inf}^2 \gg M_P^2 R_{inf} \quad , \quad \frac{M_P^2}{m^2} R_{inf}^2 \gg \xi R_{inf}^4 \quad , \quad (3)$$

where ξ is an appropriate parameter for the R^4 terms. This hierarchy has no apparent justification and a concrete answer to why the ξR^4 terms are expected to be small (therefore pose no threat) is not known. Proposals of why the hierarchy (3) is expected to hold in a supergravity framework can be found in [17, 18].

A second open issue, which is again related to the scale $m \sim 10^{-5} M_P$ is the initial conditions problem [19, 20]. If our universe exited the quantum gravity regime with an energy density $\sim M_P^4$ [21, 22], then due to the characteristic upper bound of the potential energy of the Starobinsky model $\sim 10^{-10} M_P^4$, the total energy density has to be dominated by the kinematic contribution. This leads to a need for an initial homogeneous patch of radius of the order

$$r_{init} \sim 10^3 l_P, \quad (4)$$

which means rather special initial conditions. A proposal of how this is ameliorated in a pure $R + R^2$ setup (supergravity or not), has been given in [23] which we also review here. For a review on inflationary cosmology after the release of the Planck collaboration's results, and for an approach on the initial conditions problem see also [24].

Note that the embedding of the Starobinsky model has been also studied in the old-minimal supergravity [13, 25–30], in no-scale supergravity [31–34], in the linearized non-minimal (20/20) supergravity [35], and also in a generic supergravity setup via gravitino condensates [36, 37].

2 $R + R^2$ new-minimal supergravity

The new-minimal supergravity [11] is the supersymmetric theory of the gravitational multiplet

$$e_m^a, \quad \psi_m^\alpha, \quad A_m, \quad B_{mn} \quad . \quad (5)$$

The first two fields are the vierbein and its superpartner the gravitino, a spin- $\frac{3}{2}$ Rarita-Schwinger field. The last two fields are auxiliaries. The real auxiliary vector A_m gauges the $U(1)_R$ chiral symmetry.

The auxiliary B_{mn} is a real two-form appearing only through its dual field strength H_m , which satisfies $\hat{D}^a H_a = 0$ for the supercovariant derivative \hat{D}^a .

We will use superspace techniques to guarantee that our component form Lagrangians are supersymmetric. The interested reader may consult for example [8] where a treatment of the new-minimal superspace is given. The new-minimal supergravity free Lagrangian is given by

$$\mathcal{L}_0 = -2M_P^2 \int d^4\theta EV_R. \quad (6)$$

Here V_R is the gauge multiplet of the R-symmetry, which (in the appropriate WZ gauge) contains the auxiliary fields in its vector component

$$-\frac{1}{2}[\nabla_\alpha, \bar{\nabla}_{\dot{\alpha}}]V_R| = A_{\alpha\dot{\alpha}}^- = A_{\alpha\dot{\alpha}} - 3H_{\alpha\dot{\alpha}}, \quad (7)$$

and the Ricci scalar in its highest component

$$\frac{1}{8}\nabla^\alpha\bar{\nabla}^2\nabla_\alpha V_R| = -\frac{1}{2}(R + 6H^a H_a). \quad (8)$$

The symbol E stands for the super-determinant of new-minimal supergravity. The bosonic sector of Lagrangian (6) is

$$\mathcal{L}_0 = M_P^2 e \left(\frac{1}{2}R + 2A_a H^a - 3H_a H^a \right). \quad (9)$$

It is well known from linearized supergravity [14] that the R^2 term is accommodated inside

$$\mathcal{L}_{R^2} = \frac{\alpha}{4} \int d^2\theta \mathcal{E} W^2(V_R) + c.c. \quad (10)$$

with the standard definition of the field strength

$$W_\alpha(V_R) = -\frac{1}{4}\bar{\nabla}^2\nabla_\alpha V_R, \quad (11)$$

and \mathcal{E} the chiral density. In component form, the bosonic sector of (10) reads

$$e^{-1}\mathcal{L}_{R^2} = \frac{\alpha}{8} (R + 6H^2)^2 - \frac{\alpha}{4} F^2(A^-). \quad (12)$$

The Starobinsky model of inflation in new-minimal supergravity (now we set $M_P = 1$) is [13]

$$\mathcal{L} = -2 \int d^4\theta EV_R + \frac{\alpha}{4} \int d^2\theta \mathcal{E} W^2(V_R) + c.c.$$

with bosonic sector

$$e^{-1}\mathcal{L} = \frac{1}{2}R + 2A_a H^a - 3H_a H^a + \frac{\alpha}{8} (R + 6H^2)^2 - \frac{\alpha}{4} F^2(A^-). \quad (13)$$

Indeed, theory (13) describes $R + R^2$, but the curvature terms are mixed with the auxiliary field H^a .

To find the theory in the Einstein frame we proceed to integrating out the auxiliary fields. The Lagrangian (13) is classically equivalent to

$$e^{-1}\mathcal{L} = \frac{1}{2}R + 2A_a H^a - 3H_a H^a - 2H^m \partial_m X + \frac{\alpha}{8} \Psi (R + 6H^2) - \frac{\alpha}{32} \Psi^2 - \frac{\alpha}{4} F^2(A^-), \quad (14)$$

where now H_m is an unconstrained real vector. Indeed, by integrating out the real scalar X , it imposes the appropriate constraint on H_m , and by integrating out Y we get (13). Now we redefine the auxiliary A_m as

$$\mathcal{V}_m = A_m - 3H_m - \partial_m X, \quad (15)$$

and we find

$$e^{-1}\mathcal{L} = \frac{1}{2} \left(1 + \frac{\alpha}{4}\Psi\right) R - \frac{\alpha}{4}F^2(\mathcal{V}) + 2\mathcal{V}_a H^a + 3 \left(1 + \frac{\alpha}{4}\Psi\right) H^2 - \frac{\alpha}{32}\Psi^2. \quad (16)$$

The auxiliary H_m has become quadratic and after it is integrated out we have

$$e^{-1}\mathcal{L} = \frac{1}{2} \left(1 + \frac{\alpha}{4}\Psi\right) R - \frac{\alpha}{4}F^2(\mathcal{V}) - \frac{\alpha}{32}\Psi^2 - \frac{1}{3} \frac{\mathcal{V}^2}{\left(1 + \frac{\alpha}{4}\Psi\right)}. \quad (17)$$

Note that the original A_m not only has become propagating, but it has also become massive. After rescaling the theory to go to the Einstein frame by a conformal transformation

$$e_m^a \rightarrow \frac{1}{\sqrt{1 + \frac{\alpha}{4}\Psi}} e_m^a, \quad (18)$$

we have (for $\Psi \rightarrow \Psi/\alpha$ and $\mathcal{V} \rightarrow \mathcal{V}/\sqrt{\alpha}$)

$$e^{-1}\mathcal{L} = \frac{1}{2}R - \frac{1}{4}F^2(\mathcal{V}) - \frac{3}{64(1 + \frac{1}{4}\Psi)^2} \partial\Psi\partial\Psi - \frac{1}{32\alpha}\Psi^2 \frac{1}{(1 + \frac{1}{4}\Psi)^2} - \frac{1}{3\alpha} \frac{\mathcal{V}^2}{(1 + \frac{1}{4}\Psi)^2}. \quad (19)$$

Finally, for

$$\phi = \sqrt{\frac{3}{2}} \ln \left(1 + \frac{1}{4}\Psi\right), \quad (20)$$

we have (for $\frac{1}{\alpha} = 9g^2$)

$$e^{-1}\mathcal{L} = \frac{1}{2}R - \frac{1}{4}F^2(\mathcal{V}) - \frac{1}{2}\partial\phi\partial\phi - \frac{9g^2}{2} \left(1 - e^{-\sqrt{\frac{2}{3}}\phi}\right)^2 - 3g^2 e^{-2\sqrt{\frac{2}{3}}\phi} \mathcal{V}^2. \quad (21)$$

This is the dual form of the Starobinsky model. Here we have reproduced it in a new-minimal supergravity framework [13]. Notice that there is only one real propagating scalar field, and a massive vector. From (21) one can calculate the slow-roll parameters and verify that the model is in perfect agreement with the Planck data [1].

3 Open issues in the Starobinsky model

In this part we review some known open issues of the Starobinsky model.

3.1 Higher order corrections

On top of the $R + R^2$ theory one could ask what is the impact of the higher order corrections. We will consider here the R^4 terms. The superspace Lagrangian for R^4 has the form [13]

$$\mathcal{L}_{R^4} = 16\xi \int d^4\theta E W^2(V_R) \bar{W}^2(V_R). \quad (22)$$

The full Lagrangian including the R^4 terms reads

$$\mathcal{L} = -2 \int d^4\theta E V_R + \left\{ \frac{\alpha}{4} \int d^2\theta \mathcal{E} W^2(V_R) + c.c. \right\} + 16\xi \int d^4\theta E W^2(V_R) \bar{W}^2(V_R). \quad (23)$$

During inflation only the curvature terms contribute, therefore we can work with the Lagrangian

$$e^{-1}\mathcal{L} = \frac{1}{2}R + \frac{\alpha}{8}R^2 + \xi R^4. \quad (24)$$

The bosonic terms that we have ignored in writing (24) would only contribute to the vector sector in the dual description (see [13, 15, 18]). We can then rewrite the theory with the use of a Lagrange multiplier Z as

$$e^{-1}\mathcal{L} = \left(\frac{1}{2} + Z\right)R + \frac{\alpha}{8}Y^2 + \xi Y^4 - ZY. \quad (25)$$

Indeed, by integrating out Z we find $Y = R$ and we get (24). Now we proceed in the other direction and we integrate out Y . The equation of motion for Y gives

$$Y^3 + \frac{\alpha}{16\xi}Y - \frac{Z}{4\xi} = 0, \quad (26)$$

which can be solved as

$$Y(Z) = \frac{1}{3} \left(\frac{27}{8\xi}Z + \frac{1}{2} \sqrt{\left(\frac{27}{4\xi}Z\right)^2 + 4\left(\frac{3\alpha}{16\xi}\right)^3} \right)^{\frac{1}{3}} - \frac{\alpha}{16\xi} \left(\frac{27}{8\xi}Z + \frac{1}{2} \sqrt{\left(\frac{27}{4\xi}Z\right)^2 + 4\left(\frac{3\alpha}{16\xi}\right)^3} \right)^{-\frac{1}{3}}. \quad (27)$$

After integrating out Y , rescaling the metric and redefining Z we find

$$e^{-1}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\partial\phi\partial\phi - V(\phi), \quad (28)$$

with scalar potential

$$V(\phi) = \frac{3\xi Y^4(Z) + \frac{\alpha}{8}Y^2(Z)}{4(Z + \frac{1}{2})^2} \Big|_{Z=\frac{1}{2}e^{\sqrt{\frac{2}{3}}\phi} - \frac{1}{2}}. \quad (29)$$

The plot of the scalar potential (29) can be seen in Figure 1. It is easy to see that for small ξ values inflation is not ruined, but larger ξ values may pose a threat [13, 18].

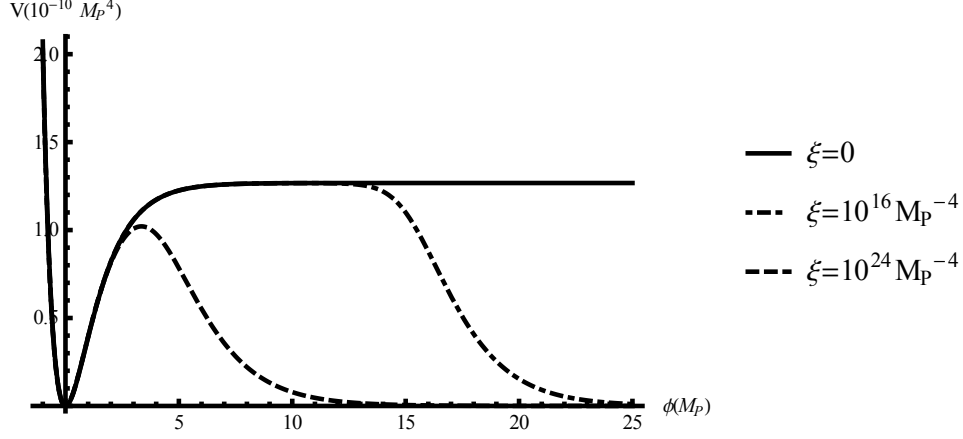


Figure 1: The potential for the Starobinsky model in the dual description, including R^4 terms parameterized by ξ . Here we have set $\alpha \sim 0.4 \times 10^{10} M_P^2$ as constrained by the Planck data. One can see the characteristic plateau of the Starobinsky model (for $\xi = 0$) at $V_{inf} \sim 1.3 \times 10^{-10} M_P^4$.

3.2 Initial conditions problem

The common lore is that inflation started when our universe exited the quantum gravity regime with energy densities [21, 22]

$$\frac{1}{2}(\partial\phi)^2 \lesssim V(\phi) \sim \frac{1}{2}\rho_{tot} \sim \frac{1}{2}M_P^4. \quad (30)$$

In this case the potential energy dominates the total energy density and the accelerated expansion starts even for a fundamentally small initial patch of Planck length radius l_P . The essential ingredient for inflation in this setup is the existence of a nearly constant event horizon distance, also of size $\sim l_P$. The importance of the existence of the event horizon is that it protects the initial smooth patch from the outside inhomogeneities with nonzero gradients. If there was no event horizon, these inhomogeneities would infest the initial smooth patch and spoil inflation [19].

For the Starobinsky inflation we have

$$V_{inf} = \frac{3}{4}m^2 M_P^2 \sim 10^{-10} M_P^4 \ll M_P^4. \quad (31)$$

For the total energy density when our universe exits the quantum gravity regime to be $\sim M_P^4$, one has to assume

$$V(\phi) \ll \frac{1}{2}\dot{\phi}^2 \sim \rho_{tot}. \quad (32)$$

In other words, that a kinematic energy domination regime preceded the inflationary phase. In such a case, $V(\phi) \ll \frac{1}{2}\dot{\phi}^2 \sim \rho_{tot}$, the scale factor grows like $t^{1/3}$ until the domination of the plateau potential yielding an event horizon of size

$$d_{\text{event}}(t \sim t_P) \sim 10^3 H_P^{-1}, \quad (33)$$

where $H_P^{-1} \equiv \sqrt{3}l_P$. Hence, one has to expel the density inhomogeneities at least 10^3 Hubble scales further if the Universe has emerged from the Planck densities. The corresponding initially homogeneous volume is at least 10^9 times bigger than H_P^{-3} which means that, initially, one billion causally disconnected regions were much similar without any dynamical reason.

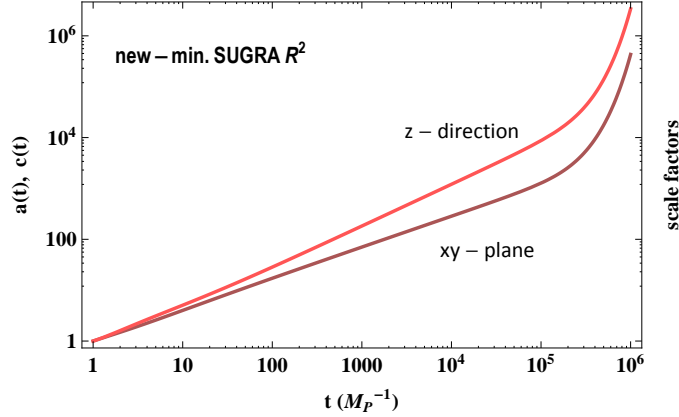


Figure 2: The evolution of the scale factors. The anisotropic expansion of the Universe is manifest. After the onset of inflation t_{INF} the scale factors evolve similarly and the anisotropy gets diluted.

As we have seen the embedding of the Starobinsky model in the new-minimal supergravity framework has given rise to an additional propagating massive vector field. Pursuing a minimal setup it has been proposed in [23] that the existence of this vector field can help to ameliorate the initial conditions problem. Indeed, one can accomplish an equipartition of the energy density

$$\frac{1}{2}\rho_{kin} \sim \frac{1}{2}\rho_{pot} \sim \frac{1}{2}M_P^4, \quad (34)$$

by invoking non vanishing values not only for the scalar field but also for the components of the vector field \mathcal{V}_m .

One can choose the gauge

$$\mathcal{V}_0 = 0, \quad (35)$$

and take the z -spatial axis parallel to the direction of the vector

$$\mathcal{V}_i = \mathcal{A}_z(t)\delta_i^z. \quad (36)$$

By giving to the vector a non-vanishing value a spatial direction is singled out which we have identified with the z -axis. This implies that the metric will be described by two scale factors

$$ds^2 = -dt^2 + a^2(t)[dx^2 + dy^2] + c^2(t)dz^2, \quad (37)$$

hence, an anisotropy is created.

A numerical solution to the system of the equations and the evolution of the two scale factors, for $\rho_{kin,init} = 0.5V_{init}$ can be seen in Figure 2 and Figure 3. Accordingly, the event horizon distances change in the z -direction and the $x-y$ plane. The initial condition problem is indeed relaxed, but still a large homogeneous initial patch is required. For a complete analysis the reader is referred to [23], where also a discussion on the old-minimal supergravity embedding can be found.

It is worth mentioning that the initial conditions for Starobinsky inflation, in a pure gravitational setup, need a minimum amount of tuning for the case of open universe. Indeed there, the volume of the initial homogeneous patch is more than 10^6 times smaller than the volume needed for closed or flat universe. For a complete discussion see [23].

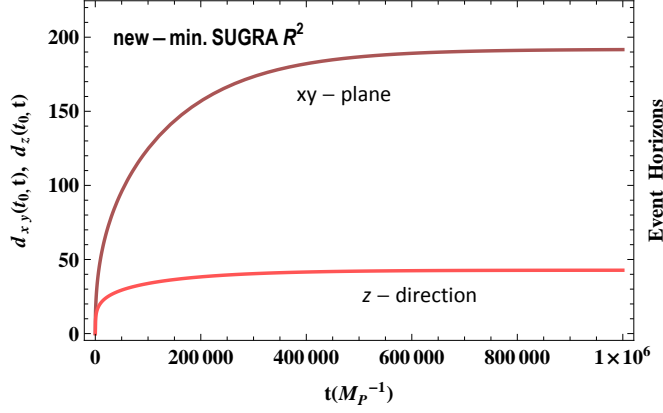


Figure 3: The evolution of the event horizons. The anisotropic expansion of the Universe implies that the event horizon distance is respectively anisotropic. The event horizon distance is in $H^{-1}(t_P) = \sqrt{3}l_P$ units.

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